Three-Quark Wave Function of Nucleon in the Quasipotential Approach

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Abstract

The structure of the nucleon wave function as a bound system of the constituent quarks was considered in framework of the quasipotential method of description of the bound states with a fixed number of particles. In the impulse approximation the wave function is reduced to the three-quark component of the vector state in the Fock-momentum space. Spin structure of the wave function was studied by the decomposition method of the irreducible representations product of inhomogeneous Lorentz group. Physical distinction of SU(6) symmetric solution is determined by uniqueness of this solution in the nonrelativistic limit. The effective mass approximation for the relativistic theory was numerically studied, in which dispersion of the average quark momentum is small as compare to a big average quark momentum. Relativistic generalization of the nonrelativistic three-particle oscillator was proposed.

Introduction.

The two approaches have a leading position among others to solve a question about the construction of the relativistic many particle wave function (WF). The first approach based on many-time 4-dimensional formalizm and its various 3-dimentional reductions (quasipotential (QP) formulations), in which WF is defined on equal-time surface in the center-of-mass system or in laboratory system. The second one is formulated in light front formalism [1]. Vacuum fluctuations are absent in the light front formalism and this allows to apply the fixed number particle approximation. The light front formalism is more suitable for description at high energies [2].

It may be expected that the first approach listed above is preferable at low and medium energies. The aim of this work is to demonstrate this property by using example of description of nucleon in the quark model in the Logunov-Tavkhelidze QP approach [3], in covariant formulation [4]. The vacuum fluctuations, which are called the main defect of this method [5], may be suppressed at low energies. One task for developing approach consists of the answer to the question: in what region mechanizm of constituent quark may be applied for the nucleon, and that must be solved by using of the experimental data.

The fixed number particles assumption may be formally expressed as a zero-order interaction in quark operators in the Bethe-Salpeter wave function. This allows one to reduce QP WF to the decomposition component of the vector state in the Fock-momentum space. Then it is possible to apply an apparatus of decomposition of the product of irreducible representations of the Lorentz-group [6, 7], which are analogous of the Clebsh-Gordan decomposition in quantum mechanics. Thus the QP method is the analogy of quantum mechanics description, which was fruitfully used by many authors. In the third section of the article this property of the QP approach is used to obtain a solution of the three particle relativistic oscillator.

1. Three quark wave function of nucleon in quasipotential approach.

Let us suppose that the Fock-momentum space for nucleon has a 3-quark basis and the nucleon vector state in this space decomposes as:

$$\mid KJ\lambda_J T\lambda_T\rangle = \frac{1}{(2\pi)^9}\int(\prod_{k=1}^{3} d\Omega_{\mathbf{p}_k}) \mid \{\mathbf{p}_k s_k\lambda_{s_k} t_k\lambda_{t_k}\}\rangle\langle\{\mathbf{p}_k s_k\lambda_{s_k} t_k\lambda_{t_k}\} \mid KJ\lambda_J T\lambda_T\rangle. \quad (1)$$

Orthonormalization conditions for the nucleon and quark vector states are following:

$$\langle K'J'\lambda'_J T'\lambda'_T \mid KJ\lambda_J T\lambda_T\rangle = (2\pi)^3\delta^3(\mathbf{K}'-\mathbf{K})\delta_{\lambda'_J\lambda_J}\delta_{\lambda'_T\lambda_T},$$

$$\langle \mathbf{p}'_k s_k\lambda'_{s_k} t_k\lambda'_{t_k} \mid \mathbf{p}_k s_k\lambda_{s_k} t_k\lambda_{t_k}\rangle = (2\pi)^3 E_{\mathbf{p}_k}\delta^3(\mathbf{p}'_k-\mathbf{p}_k)\delta_{\lambda'_{s_k}\lambda_{s_k}}\delta_{\lambda'_{t_k}\lambda_{t_k}},$$

$$\bar{u}^{\lambda_{s_k}}_{\alpha_k}(\mathbf{p})u^{\lambda_{s_k}}_{\alpha_k}(\mathbf{p}) = m \quad \text{(there is no sum over } \lambda_{s_k}). \quad (2)$$

Here and further $\{\mathbf{p}_k s_k\lambda_{s_k} t_k\lambda_{t_k}\}$ means a set of values with indices k=1,2,3; $d\Omega_{\mathbf{p}_k} = d\mathbf{p}_k/E_{\mathbf{p}_k}$; $\mathbf{p}_k,\ m$ are momentum and mass of quark; $K = (\sqrt{\mathbf{K}^2+M^2}, \mathbf{K})$, M is the nucleon mass; $J,\ s_k$ are spins and $T,\ t_k$ are isospins of nucleon and constinuent quarks; λ_x is the third projection of x. In this section the isospin indices will be omitted. We assume the sum over the repeated spin and isospin indices (if we don't indicate espessially).

In the quasipotential approach let us consider a covariant projection of the Bethe-Salpeter wave function on equal-times surface in the center-of-mass system [3, 4]:

$$\Psi^{\{\alpha_k\}}_{KJ\lambda_J}(\{x_k\}) = \Psi^{BS\{\alpha_k\}}_{KJ\lambda_J}(\{x_k\})\delta(n_K(x_1-x_2))\delta(n_K(x_1-x_3)), \quad (3)$$

where $\Psi^{BS\{\alpha_k\}}_{KJ\lambda_J}(\{x_k\}) = \langle 0 \mid T\left[\phi^{(1)}_{\alpha_1}(x_1)\phi^{(2)}_{\alpha_2}(x_2)\phi^{(3)}_{\alpha_3}(x_3)\right] \mid KJ\lambda_J\rangle$ is the Bethe-Salpeter wave function, $(n_K)_\mu = K_\mu/\sqrt{K^2}$ is the unit 4-momentum. δ-functions make the times to be equal covariantly in the c.m.s. Using a property of translational invariance, it is possible to separate motion of center-of-mass system and then the Fourier-representation of the wave function (3) is given by [4], notation $\sum_{k=1}^{3}\overset{o}{p}_{k0}=\overset{o}{P}_0$ is introduced:

$$\Psi^{\{\alpha_k\}}_{KJ\lambda_J}(\{p_k\}) = \left[\prod_{k=1}^{3} S_{\alpha_k\beta_k}(L_K)\right](2\pi)^4\delta^{(1)}(\overset{o}{P}_0 - M)\delta^{(3)}(\sum_{k=1}^{3}\overset{o}{\mathbf{p}}_k)\Phi^{\beta_k}_{MJ\lambda_J}(\overset{o}{\mathbf{p}}_k), \quad (4)$$

where $S_{\alpha_k\beta_k}(L_K)$ is a matrix of the Lorentz-boost representation L_K, $L_K^{-1}K \equiv\overset{o}{K}= (M,\mathbf{0})$, $\overset{o}{p}_k= L_K^{-1}p_k$. In (4) the wave function depends on two independent 3-momenta. For the sake of symmetry we keep all the momenta assuming their relation $\sum_{k=1}^{3}\overset{o}{\mathbf{p}}_k= 0$. The expression for Φ has the form:

$$\Phi^{\beta_k}_{MJ\lambda_J}(\overset{o}{\mathbf{p}}_k) = \int[\prod_{k=2}^{3} d\overset{o}{y}_k \exp(-i\overset{o}{\mathbf{p}}_k\overset{o}{y}_k)]\langle 0 \mid \phi^{(1)}_{\beta_1}(0)\phi^{(2)}_{\beta_2}(0,\overset{o}{y}_2)\phi^{(3)}_{\beta_3}(0,\overset{o}{y}_3) \mid M\overset{o}{\mathbf{K}} J\lambda_J\rangle. \quad (5)$$

In the quantum-field interpretation the fixed number particles assumption is expressed in replacement of the Heizenberg field operators to the Dirac ones (zero-order interaction in field operators). Bound effects are contained in the Heizenberg vector state of the bound system. We have applied this assumption in (5) and substituted it in (4) and as a result the wave function (4) is expressed through the decomposition component (1) in the form:

$$\Psi^{\{\alpha_k\}}_{KJ\lambda_J}(\{p_k\}) = (2\pi)\delta^{(1)}(\overset{o}{P}_0 - M)\left[\prod_{k=1}^{3} u^{(\lambda'_{s_k})}_{\alpha_k}(\mathbf{p}_k)\overset{+}{D}{}^{1/2}_{\lambda'_{s_k}\lambda_{s_k}}(L_K^{-1},p_k)\right]\langle\{\overset{o}{\mathbf{p}}_k\ s_k\lambda_{s_k}\} \mid M\overset{o}{\mathbf{K}} J\lambda_J\rangle. \quad (6)$$

For obtaining (6) we have used the relations:

$$S_{ri}(L_K)u_i^{(\lambda_{s_k})}(\overset{o}{\mathbf{p}}_k) = u_r^{(\lambda'_{s_k})}(\mathbf{p}_k)\ \overset{+}{D}{}^{1/2}_{\lambda'_{s_k}\lambda_{s_k}}(L_K^{-1}, p_k),$$

$$\hat{U}(L_K^{-1})|\mathbf{p}_k s_k \lambda_{s_k}\rangle = |\ \overset{o}{\mathbf{p}}_k\ s_k \lambda'_{s_k}\rangle D^{1/2}_{\lambda'_{s_k}\lambda_{s_k}}(L_K^{-1}, p_k), \tag{7}$$

where $D^{1/2}(L_K^{-1}, p_k)$ is the Wigner spin rotation matrix, which is defined as a 2×2 representation of the 3-dimensional rotation $R^W_{(L_K^{-1}, p)} = L^{-1}_{\overset{o}{\mathbf{p}}} L_K^{-1} L_p$ [6, 7, 8, 9]. It has an explicit form [6]:

$$D(L_K^{-1}, p_k) = \frac{(E_{\mathbf{p}_k} + m)(E_K^M + M) - (\sigma \mathbf{K})(\sigma \mathbf{p}_k)}{\sqrt{2(E_{\mathbf{p}_k} + m)(E_K^M + M)(E_{\mathbf{K}} E_{\mathbf{p}_k} - \mathbf{K}\mathbf{p}_k + mM)}}$$

We multiply (6) by $\prod_{k=1}^3 (1/m)\bar{u}_{\alpha_k}^{(\lambda_{s_k})}(\mathbf{p}_k)$ from left-handed side and sum the result over the spinor index:

$$\Psi^{(+)}_{KJ\lambda_J\{s_k\lambda_{s_k}\}} = \left[\prod_{k=1}^3 (1/m)\bar{u}_{\alpha_k}^{(\lambda_{s_k})}(\mathbf{p}_k)\right] \Psi^{\{\alpha_k\}}_{KJ\lambda_J}(\{p_k\}) = (2\pi)\delta^{(1)}(\overset{o}{P}_0 - M)\langle\{\mathbf{p}_k s_k \lambda_{s_k}\} \mid KJ\lambda_J\rangle. \tag{8}$$

The function (8) is named a wave function projected onto positive frequency states [4]. Taking into account (4) and (6) we obtain:

$$\langle\{\mathbf{p}_k s_k \lambda_{s_k}\} \mid KJ\lambda_J\rangle. = (2\pi)^3 \delta^{(3)}(\sum_{k=1}^3 \overset{o}{\mathbf{p}}_k) \left[\prod_{k=1}^3 \overset{+}{D}{}^{1/2}_{\lambda'_{s_k}\lambda_{s_k}}(L_K^{-1}, p_k)\right] \Phi^{(+)}_{MJ\lambda_J\{s_k\lambda'_{s_k}\}}(\{\overset{o}{\mathbf{p}}_k\}), \tag{9}$$

where $\Phi^{(+)}_{MJ\lambda_J\{s_k\lambda'_{s_k}\}}$ is related with $\Phi^{\alpha_k}_{MJ\lambda_J}$,, defined in (4)), via:

$$\Phi^{(+)}_{MJ\lambda_J\{s_k\lambda'_{s_k}\}}(\{\overset{o}{\mathbf{p}}_k\}) = \left[\prod_{k=1}^3 (1/m)\bar{u}_{\alpha_k}^{(\lambda_{s_k})}(\mathbf{p}_k)\right] \Phi^{\alpha_l}_{MJ\lambda_J}(\{\overset{o}{\mathbf{p}}_k\}). \tag{10}$$

Using the method of the two-time Green function [3, 4] it is possible to obtain quasipotential equation for function $\Phi^{(+)}$:

$$\frac{1}{(2\pi)^6 E_{\overset{o}{\mathbf{p}}_3}}(M_0 - M)\Phi^{(+)}_{MJ\lambda_J\{s_k\lambda_{s_k}\}}(\{\overset{o}{\mathbf{p}}_k\}) = \int d\Omega_{\overset{o}{\mathbf{p}}{}'_1} d\Omega_{\overset{o}{\mathbf{p}}{}'_2} V_{\{\lambda_{s_k}\lambda'_{s_k}\}}(M, \{\overset{o}{\mathbf{p}}_k \mid \overset{o}{\mathbf{p}}{}'_k\})\Phi^{(+)}_{MJ\lambda_J\{s_k\lambda'_{s_k}\}}(\{\overset{o}{\mathbf{p}}{}'_k\}). \tag{11}$$

In the impulse approximation the quasipotential $V_{\{\lambda_{s_k}\lambda'_{s_k}\}}(M, \{\overset{o}{\mathbf{p}}_k \mid \overset{o}{\mathbf{p}}{}'_k\})$ may be independent of the energy, because of the nucleon mass in the equation iteration (11) may be replaced in quasipotential by sum of the free quark energy in c.m.s. $M_0(\{\overset{o}{\mathbf{p}}_k\}) = \sum_{k=1}^3 E_{\overset{o}{\mathbf{p}}_k}$ and then the next order in the coupling constant may be neglected. The wave function normalization condition obtained by means of the Green function [4] for the quasipotential independent of energy has a symmetric form relatively the particle permutations:

$$\frac{1}{(2\pi)^6}\int d\Omega_{\overset{o}{\mathbf{p}}_1} d\Omega_{\overset{o}{\mathbf{p}}_2} \mid \Phi^{+}_{MJ\lambda_J\{s_k\lambda_{s_k}\}}(\{\overset{o}{\mathbf{p}}_k\}) \mid^2 \frac{1}{E_{\overset{o}{\mathbf{p}}_3}} = 1. \tag{12}$$

2. Nucleon spin wave function.

Now let us use the general method to construct the state with a definite momentum in the system of free particles to study spin dependence of QP WF by using connection of QP WF with a decomposition component of the nucleon vector state in the Fock-momentum space (9). In the center-of mass system formula (9) has the form:

$$\langle\{\overset{o}{\mathbf{p}}_k\ s_k\lambda_{s_k}\}|M\ \overset{o}{\mathbf{K}}\ J\lambda_J\rangle = (2\pi)^3\delta^{(3)}(\sum_{i=1}^{3}\overset{o}{\mathbf{p}}_i)\Phi^{(+)}_{J\lambda_J\{s_k\lambda_{s_k}\}}(\{\overset{o}{\mathbf{p}}_k\}). \tag{13}$$

Now we decompose $|\{\overset{o}{\mathbf{p}}_k\ s_k\lambda_{s_k}\}\rangle$ over free particle states with a definite momentum in the c.m.s. $|M_0(\{\overset{o}{\mathbf{p}}_k\})\sum_k \overset{o}{\mathbf{p}}_k\ J\lambda_J(\ldots)\rangle$. The dots mean other observables from the complete set which will be indicated below. Its choise depends on the ways of summing spins and orbital angular momenta. Further we use the method of work [6] to decompose the direct product of irreducible representations of the inhomogeneous Lorentz group.

In case of the two free particle system in its c.m.s. with 3-momentum $\overset{\not{o}}{\mathbf{p}}_1$ and $\overset{\not{o}}{\mathbf{p}}_2$ ($\overset{\not{o}}{\mathbf{p}}_{12}\equiv\overset{\not{o}}{\mathbf{p}}_1+\overset{\not{o}}{\mathbf{p}}_2=0$) and with the invariant system [1+2] mass $M_{12}\equiv E_{\overset{\not{o}}{\mathbf{p}}_1}+E_{\overset{\not{o}}{\mathbf{p}}_2}$, this decomposition has the following form:

$$|\ \overset{\not{o}}{\mathbf{p}}_1\ s_1\lambda_{s_1}\ \overset{\not{o}}{\mathbf{p}}_2\ s_2\lambda_{s_2}\rangle = |M_{12}\ \overset{\not{o}}{\mathbf{p}}_{12} j\lambda_j(ls)\rangle\langle j\lambda_j|l\lambda_l s\lambda_s\rangle Y_{l\lambda_l}(\hat{\overset{\not{o}}{\mathbf{p}}_1})\langle s\lambda_s|s_1\lambda_{s_1}s_2\lambda_{s_2}\rangle. \tag{14}$$

Here $\hat{\overset{\not{o}}{\mathbf{p}}_1}$ is angle variables of momentum $\overset{\not{o}}{\mathbf{p}}_1$, $Y_{l\lambda_l}$ is spherical harmonics, l is the relative orbital momentum of the system [1+2], $s=0,1$ is spin, j is the total spin of system [1+2]. Going from the rest system [1+2] to c.m.s. of nucleon, we have from (14):

$$|E_{\overset{o}{\mathbf{p}}_1}+E_{\overset{o}{\mathbf{p}}_2}\ \overset{o}{\mathbf{p}}_{12}\ j\lambda_j(ls)\rangle = \hat{U}(L_{\overset{o}{\mathbf{p}}_{12}})|M_{12}\ \overset{\not{o}}{\mathbf{p}}_{12}\ =\mathbf{0}j\lambda_j(ls)\rangle, \tag{15}$$

where $\overset{o}{p}_{12}=\overset{o}{p}_1+\overset{o}{p}_2$, $\overset{o}{p}_k=(L_{\overset{\not{o}}{p}_{12}}\ \overset{\not{o}}{p}_k)=(L_K^{-1}p_k)$. If we introduce the notation $y^{[12]l}_{j\lambda_j s\lambda_s}(\hat{\overset{\not{o}}{\mathbf{p}}_1})\equiv$ $\langle j\lambda_j|l\lambda_l s\lambda_s\rangle Y_{l\lambda_l}(\hat{\overset{\not{o}}{\mathbf{p}}_1})$, then applying (7) we obtain:

$$\begin{aligned}|\ \overset{o}{\mathbf{p}}_1\ s_1\lambda_{s_1}\ \overset{o}{\mathbf{p}}_2\ s_2\lambda_{s_2}\rangle &= |E_{\overset{o}{\mathbf{p}}_1}+E_{\overset{o}{\mathbf{p}}_2}\ \overset{o}{\mathbf{p}}_{12}\ j\lambda_j(ls)\rangle y^{[12]l}_{j\lambda_j s\lambda_s}(\hat{\overset{\not{o}}{\mathbf{p}}_1})\times\\ &\times\langle s\lambda_s|s_1\lambda'_{s_1}s_2\lambda'_{s_2}\rangle\ \overset{+}{D}{}^{(1/2)}_{\lambda'_{s_1}\lambda_{s_1}}(L_{\overset{o}{\mathbf{p}}_{12}},\overset{\not{o}}{\mathbf{p}}_1)\ \overset{+}{D}{}^{(1/2)}_{\lambda'_{s_2}\lambda_{s_2}}(L_{\overset{o}{\mathbf{p}}_{12}},\overset{\not{o}}{\mathbf{p}}_2).\end{aligned} \tag{16}$$

Multiplying (16) on the vector state of the third quark, we have obtained:

$$\begin{aligned}|\{\overset{o}{\mathbf{p}}_k\ s_k\lambda_{s_k}\}\rangle &= |E_{\overset{o}{\mathbf{p}}_1}+E_{\overset{o}{\mathbf{p}}_2}\ \overset{o}{\mathbf{p}}_{12}\ j\lambda_j(ls)\ \overset{o}{\mathbf{p}}_3\ s_3\lambda_{s_3}\rangle y^{[12]l}_{j\lambda_j s\lambda_s}(\hat{\overset{\not{o}}{\mathbf{p}}_1})\times\\ &\times\langle s\lambda_s|s_1\lambda'_{s_1}s_2\lambda'_{s_2}\rangle.\ \overset{+}{D}{}^{(1/2)}_{\lambda'_{s_1}\lambda_{s_1}}(L_{\overset{o}{\mathbf{p}}_{12}},\overset{\not{o}}{\mathbf{p}}_1)\ \overset{+}{D}{}^{(1/2)}_{\lambda'_{s_2}\lambda_{s_2}}(L_{\overset{o}{\mathbf{p}}_{12}},\overset{\not{o}}{\mathbf{p}}_2).\end{aligned} \tag{17}$$

In analogy with (14) we obtain (notation $y^{[[12]3]L}_{J\lambda_J S\lambda_S}(\hat{\overset{o}{\mathbf{p}}}_3)=\langle J\lambda_J|L\lambda_L S\lambda_S\rangle Y_{L\lambda_L}(\hat{\overset{o}{\mathbf{p}}}_3)$ is introduced):

$$|E_{\overset{o}{\mathbf{p}}_1}+E_{\overset{o}{\mathbf{p}}_2}\ \overset{o}{\mathbf{p}}_{12}\ j\lambda_j(ls);\overset{o}{\mathbf{p}}_3\ s_3\lambda_{s_3}\rangle = |M_0(\{\overset{o}{\mathbf{p}}_k\})\sum_k\overset{o}{\mathbf{p}}_k\ J\lambda_J(ls,LS)\rangle y^{[[12]3]L}_{J\lambda_J S\lambda_S}(\hat{\overset{o}{\mathbf{p}}}_3)\langle S\lambda_S|j\lambda_j s_3\lambda_{s_3}\rangle, \tag{18}$$

where S is the spin of the system [[1+2]+3], obtained by summation of j and s_3; L is the orbital angular momentum of the third quark with respect to the [1+2] system, J is the total spin of the system [[1+2]+3], obtained by summmation of S and L. Thus, the desired decomposition has the form:

$$|\{\overset{o}{\mathbf{p}}_k\ s_k\lambda_k\}\rangle = |M_0(\{\overset{o}{\mathbf{p}}_k\})\sum_k \overset{o}{\mathbf{p}}_k\ J\lambda_J(ls,LS)\rangle\ y^{[[12]3]L}_{J\lambda_J S\lambda_S}(\hat{\overset{o}{\mathbf{p}}}_3) y^{[12]l}_{j\lambda_j s\lambda_s}(\hat{\overset{\not o}{\mathbf{p}}}_1\ ) \times$$
$$\times\langle S\lambda_S(js_3)|j\lambda_j(ls)s_3\lambda_{s_3}\rangle\langle s\lambda_s|s_1\lambda'_{s_1}s_2\lambda'_{s_2}\rangle\ \overset{+}{D}{}^{(1/2)}_{\lambda'_{s_1}\lambda_{s_1}}(L_{\overset{o}{\mathbf{p}}_{12}},\overset{\not o}{\mathbf{p}}_1\ )\ \overset{+}{D}{}^{(1/2)}_{\lambda'_{s_2}\lambda_{s_2}}(L_{\overset{o}{\mathbf{p}}_{12}},\overset{\not o}{\mathbf{p}}_2\ ) \quad (19)$$

We have separated motion of c.m.s. in the scalar product of the bound system vector state and the free particle vector state:

$$\langle M\ \overset{o}{\mathbf{K}}=\mathbf{0}J\lambda_J|M_0(\{\overset{o}{\mathbf{p}}_k\})\sum_k \overset{o}{\mathbf{p}}_k\ J\lambda_J(ls,LS)\rangle = (2\pi)^3\delta(\sum_k \overset{o}{\mathbf{p}}_k)A^{*(ls,LS)}_{MJ}(M_0(\{\overset{o}{\mathbf{p}}_k\})). \quad (20)$$

Independence of A on λ_J is the result the symmetry relatively the reflection. The relation (20) was obtained by using unit operator $\hat{1} = (1/V)\int_V d^3x \exp^{-\hat{\mathbf{P}}\mathbf{x}} \exp^{\hat{\mathbf{P}}\mathbf{x}}$. According to (13), (19) and (20), for relative motion WF $\Phi^{(+)}$ in full record with isospin indices we have obtained:

$$\Phi^{(+)*}_{MJ\lambda_J T\lambda_T\{s_k\lambda_{s_k}t_k\lambda_{t_k}\}}(\{\overset{o}{\mathbf{p}}_k\}) = A^{*(ls,LS)}_{MJT\lambda_T\{t_k\lambda_{t_k}\}}(M_0(\{\overset{o}{\mathbf{p}}_k\}))y^{[[12]3]L}_{J\lambda_J Sm_S}(\hat{\overset{o}{\mathbf{p}}}_3)y^{[12]l}_{j\lambda_j sm_s}(\hat{\overset{\not o}{\mathbf{p}}}_1\ ) \times$$
$$\times\langle S\lambda_S|j\lambda_j s_3\lambda_{s_3}\rangle\langle s\lambda_s|s_1\lambda'_{s_1}s_2\lambda'_{s_2}\rangle\ \overset{+}{D}{}^{(1/2)}_{\lambda'_{s_1}\lambda_{s_1}}(L_{\overset{o}{\mathbf{p}}_{12}},\overset{\not o}{\mathbf{p}}_1\ )\ \overset{+}{D}{}^{(1/2)}_{\lambda'_{s_2}\lambda_{s_2}}(L_{\overset{o}{\mathbf{p}}_{12}},\overset{\not o}{\mathbf{p}}_2\ ). \quad (21)$$

Since the contributions of higher orbital moments in nucleon are small [10], we write only the S-wave part ($L = l = 0$) omitting spin, isospin and symbols $L = l = 0$:

$$\Phi^{(+)}_{M\{\lambda_{s_k}\lambda_{t_k}\}}(\{\overset{o}{\mathbf{p}}_k\}) = D^{(1/2)}_{\lambda_{s_1}\lambda'_{s_1}}(L_{\overset{o}{\mathbf{p}}_{12}},\overset{\not o}{\mathbf{p}}_1\ )D^{(1/2)}_{\lambda_{s_2}\lambda'_{s_2}}(L_{\overset{o}{\mathbf{p}}_{12}},\overset{\not o}{\mathbf{p}}_2\ )\delta_{\lambda_{s_3}\lambda'_{s_3}}\xi^j_{\{\lambda'_{s_k}\}}A^{(j)}_{M\{\lambda_{t_k}\}}(M_0(\{\overset{o}{\mathbf{p}}_k\})), \quad (22)$$

where $\xi^j_{\{\lambda_{s_k}\}} = \sum_{j=0,1}\langle\lambda_{s_1}\lambda_{s_2}|j\lambda_j\rangle\langle j\lambda_j\lambda_{s_3}|\lambda_J\rangle$. Let us decompose functions $A^{(j)}_{M\{\lambda_{t_k}\}}$ over two isospin basic functions $\eta^\tau_{\{\lambda_{t_k}\}}$; $\tau = 0,1$ is the [1+2] system isospin : $A^{(j)}_{M\{\lambda_{t_k}\}} = B^{(j,\tau)}_M\eta^\tau_{\{\lambda_{t_k}\}}$ (sum over τ) and substitute it in (22):

$$\Phi^{(+)}_{\{\lambda_{s_k}\lambda_{t_k}\}}(\{\overset{o}{\mathbf{p}}_k\}) = \left[D^{(1/2)}_{\lambda_{s_1}\lambda'_{s_1}}(L_{\overset{o}{\mathbf{p}}_{12}},\overset{\not o}{\mathbf{p}}_1\ )D^{(1/2)}_{\lambda_{s_2}\lambda'_{s_2}}(L_{\overset{o}{\mathbf{p}}_{12}},\overset{\not o}{\mathbf{p}}_2\ )\delta_{\lambda_{s_3}\lambda'_{s_3}}\right]\xi^j_{\{\lambda'_{s_k}\}}B^{(j,\tau)}_M(M_0(\{\overset{o}{\mathbf{p}}_k\}))\eta^\tau_{\{\lambda_{t_k}\}}. \quad (23)$$

The function $\Phi^{(+)}$ (21) is symmetrical relatively the particle permutations and with antisymmetric colour fuction realizes antisymmetrical representation of the permutations group. S-wave function (23) with zero-orbital momentum of the [1+2] system and zero angular orbital momentum of the third quark with respect to the [1+2] system, is antisymmetric relatively the quark permutations. Let us suppose, that interquark interaction does not depend on the spin and isospin: $B^{(j,\tau)}_M(M_0(\{\overset{o}{\mathbf{p}}_k\})) = (2\pi)^3C^{(j,\tau)}\varphi_M(M_0(\{\overset{o}{\mathbf{p}}_k\}))/\sqrt{2}$ and we choose the normalization constants to agree with the nonrelativistic theory: $C^{(0,0)} = C^{(1,1)} = 1$, $C^{(1,0)} = C^{(0,1)} = 0$. Introducing the notation for the symmetric spin-isospin wave function $\chi_{\{\lambda_{s_k}\lambda_{t_k}\}} = \left[\xi^1_{\{\lambda_{s_k}\}}\eta^1_{\{\lambda_{t_k}\}} + \xi^0_{\{\lambda_{s_k}\}}\eta^0_{\{\lambda_{t_k}\}}\right]/\sqrt{2}$, we obtain

$$\Phi^{(+)}_{\{\lambda_{s_k}\lambda_{t_k}\}}(\{\overset{o}{\mathbf{p}}_k\}) = (2\pi)^3\left[D^{(1/2)}_{\lambda_{s_1}\lambda'_{s_1}}(L_{\overset{o}{\mathbf{p}}_{12}},\overset{\not o}{\mathbf{p}}_1\ )D^{(1/2)}_{\lambda_{s_2}\lambda'_{s_2}}(L_{\overset{o}{\mathbf{p}}_{12}},\overset{\not o}{\mathbf{p}}_2\ )\delta_{\lambda_{s_3}\lambda'_{s_3}}\right]\chi_{\{\lambda'_{s_k}\lambda_{t_k}\}}\varphi_M(M_0(\{\overset{o}{\mathbf{p}}_k\})) \quad (24)$$

Two D-matrices in (24) represent the minimal kinematic violation of the $SU(6)$ symmetry. The general expression for WF (21) indicates three ways of $SU(6)$ violations,

related to relativization of the model: (1) including the admixture of the mixed $SU(6)$ symmetry in WF, (2) taking into account the quark interaction dependence on spin and isospin, (3) and P- and D-waves.

3. Effective mass approximation and relativistic oscillator.

According to (1), (12) and (24), function φ_M satisfies the equation and normalization condition:

$$[M_0(\{\overset{o}{\mathbf{p}}_k\}) - M]\varphi_M(M_0(\{\overset{o}{\mathbf{p}}_k\})) = \int d\Omega_{\overset{o}{\mathbf{p}}{}'_1} d\Omega_{\overset{o}{\mathbf{p}}{}'_2} v(\{\overset{o}{\mathbf{p}}_k \mid \overset{o}{\mathbf{p}}{}'_k\})\varphi_M(M_0(\{\overset{o}{\mathbf{p}}_k\}))/E_{\overset{o}{\mathbf{p}}{}'_3},$$

$$\int d\Omega_{\overset{o}{\mathbf{p}}_1} d\Omega_{\overset{o}{\mathbf{p}}_2} \mid \varphi_M(M_0(\{\overset{o}{\mathbf{p}}_k\})) \mid^2 / E_{\overset{o}{\mathbf{p}}_3} = 1, \tag{25}$$

where v is a quasipotential determined by relation: $v = (2\pi)^6[\chi(DDI)V(DDI)\chi]E_{\overset{o}{\mathbf{p}}{}'_3} E_{\overset{o}{\mathbf{p}}_3}$, where (DDI) is a matrix record of factors in square brackets of expression (24).

Variables $\overset{o}{\mathbf{p}}_1$, $\overset{o}{\mathbf{p}}_2$ and $\overset{o}{\mathbf{p}}_3$ are equivalent and the choise of variables $\overset{o}{\mathbf{p}}_1$, $\overset{o}{\mathbf{p}}_2$ as independent does not change the three quark equivalent to calculate the average values. For example, the three effective quark masses, obtained by formula $m^{eff} = \sqrt{m^2 + \langle p_k^2\rangle}$ are equal ($\langle p_k^2\rangle = \int d\Omega_{\mathbf{p}_1} d\Omega_{\mathbf{p}_2} \mid \varphi_M \mid^2 p_k^2/E_{\overset{o}{\mathbf{p}}_3}$). The approximate relativistic models, using the idea of the effective quark mass, were considered in work [11, 12]. In these works the effective quark mass is introduced as a parameter instead of the quark mass. In our work the effective quark mass has been introduced as a suitable approximation.

Let us pass to the semi-momenta [13] in (25): $\pi_k = \overset{o}{\mathbf{p}}_k \sqrt{2m/(m + E_{\overset{o}{\mathbf{p}}_k})}$, $E_{\overset{o}{\mathbf{p}}_k} = m + \pi_k^2/2m$. Equation (25) takes a nonrelativistic form in terms of π_k. In equation (25) we rewrite the quasipotential as an analog of nonrelativistic oscillator:

$$\left[\pi_1^2/2m + \pi_2^2/2m + \pi_3^2/2m + 3m - M - k_o(\nabla^2_{\pi_1} + \nabla^2_{\pi_2} - \nabla_{\pi_1}\nabla_{\pi_2})\right]\varphi_M^{osc} = 0. \tag{26}$$

Using new variables $\mathbf{k} = \frac{1}{2}(\pi_1 - \pi_2)$, $\mathbf{k}' = \frac{1}{3}(\pi_1 + \pi_2 - 2\pi_3)$, $\mathbf{Q} = \pi_1 + \pi_2 + \pi_3$ and parameters in (26) $\mu = m/2$, $\mu' = 2m/3$, $\omega^2 = 3k_o/m$, we have obtained:

$$\left[3m - M + \mathbf{k}^2/2\mu + \mathbf{k}'^2/2\mu' + \mathbf{Q}^2/6m - (\mu/2)\omega^2\nabla^2_{\mathbf{k}} - (\mu'/2)\omega^2\nabla^2_{\mathbf{k}'}\right]\varphi_M^{osc} = 0. \tag{27}$$

Decomposing the quark energies in semi-momenta in the m^{eff} neighborhood and taking into account the condition $\sum_{k=1}^3 \overset{o}{\mathbf{p}}_k = 0$, it is possible to show that term $\mathbf{Q}^2/6m$ has a small contribution into the energy, if value $\delta_E \equiv m^{eff} - \langle E_{p_k}\rangle$ is small (the effective mass aproximation). In the zero-order approximation relatively the $\mathbf{Q}^2/6m$, we obtain a solution of equation (27) for the ground state ($\lambda^2 = \gamma^2/2$, $\lambda'^2 = 2/3\gamma^2$, $\gamma^2 = m\omega$): $\varphi_M^{osc} \approx \exp(-k^2/2\lambda^2 - k'^2/2\lambda'^2)$. Taking into account the effective mass approximation, this solution is reduced to the form:

$$\varphi_M^{osc} \approx N\exp[-m(M_0 - 3m)/\gamma^2]. \tag{28}$$

Numerical calculations with the relativistic oscillator WF (28) gives $m^{eff}/m \in [1.08, 1.74]$ for $\gamma/m \in [0.4, 1.1]$. In this case δ_E is positive and is in limits $[0.002, 0.009]m^{eff}$. Smallness of δ_E means the smallness of the momentum module dispersion. Indeed, in the ground state $\langle \mathbf{p}_k^2 - \langle \mathbf{p}_k^2\rangle\rangle = 0$ and therefore $\delta_E \approx \sigma^2_{\mathbf{p}_k^2}/8(m^{eff})^3$, where $\sigma^2_{\mathbf{p}_k^2} = \langle[\mathbf{p}_k^2 - \langle\mathbf{p}_k^2\rangle]^2\rangle$.

We note that, WF in form $\exp[-M_0^2/\gamma^2]$, applied in many works, is different from (28) by the order of M_0 and has different asymptotic behaviour for high momenta.

Conclusion.

In the impulse approximation, represented by the zero-order field operators in the Bethe-Salpeter WF, the quasipotential wave function reduces to the three-quark component of the vector state decomposition of the bound system in the Fock-momentum space. This allows us to apply the standart method for the decomposition of the irreducible representations product of the inhomogeneous Lorentz-group over states with the definite momentum to analyse of the QP WF structure. The physical preference of the $SU(6)$-symmetric solution is determined by its uniqueness in the nonreletivistic limit.

The model of the relativistic three-particle oscillator being the direct generalization of the nonrelativistic oscillator is proposed. Numerically it was shown that the effective mass approximation may be applyed in the wide region of the oscillator parameters, in which the ratio of the momentum dispersion to the average value of the quark momentum module is small.

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